प्राकृतिक एवं भौतकीय विज्ञान शोध पत्रिका

Journal of Natural & Physical Scienes

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OPTICAL AND STRUCTURAL PROPERTIES OF BORON DOPED ZNO THIN FILMS PREPARED BY SOL-GEL TECHNIQUE

L.P. Purohit* , Vinod Kumar* and R. Kumar *

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ABSTRACT

Boron doped ZnO films have been prepared on glass substrates by the sol gel spin coating method. Zinc acetate solutions of 0.2 M in methanol stabilized by monoethanolamine and doped with boron tri methyl borate were used. Doping concentration of boron is fixed 0.2 at.%. After deposition samples were air annealed at 450°C for 1 h to improve their optical properties. X- ray diffraction shows that all films are oriented along the c-axis direction of the hexagonal crystal structure. The average optical transmittance of all films is higher than 85%. The effect of boron doping in band gap has been also measured.

Key words :- Zinc oxide, Sol-gel, Boron, Optical properties, Structural properties

INTRODUCTION

Zinc oxide (ZnO) is an II-VI semiconductor, highly transparent (85-95%) in the visible region with a wide and direct band-gap of about 3.3eV [5] at room temperature and a high exciton binding energy of 60 meV. Thin films of undoped and doped ZnO are utilized for a wide variety of electronic and opto-electronic applications, such as transparent conducting electrodes [12], heat mirrors [7] and surface acoustic wave devices [2]. Transparent conducting oxides (TCO) have a range of useful applications such as transparent electrodes in optoelectronic devices and solar cells.

Boron doped ZnO thin films were prepared by metal organic chemical vapor deposition [8], sputtering [3,6,9] and laser deposition [11]. However, boron doped ZnO thin films prepared by sol-gel method [13] are rare. The sol-gel method offers many advantages such as highly homogeneous thin films, large area coating, and absence of the need for vacuum, low cost and high flexibility.

In this paper structural and optical properties of sol-gel derived boron doped ZnO films using spin coating techniques has been investigated.

EXPERIMENTAL DETAILS

Boron doped ZnO (ZnO: B) thin films were prepared by sol gel method using spin coating techniques. The starting material for boron doped ZnO thin films was zinc acetate dihydrate [Zn $(CH_3COO)_2 \cdot 2H_2O$]. The dopant source of boron was Tri methyl borate (99.999%, high purity chemicals). Methanol was used as an alcoholic medium in this processing. Monoethanolamine

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* Department of Physics, Gurukula Kangri University, Haridwar - 249 404 Email - lppurohit@gmail.com

(MEA) is used as a complexing agent to keep the metal ions in homogeneous solution.

The Zn precursor solution was prepared by dissolving zinc acetate dihydrate in methanol so as to prepare concentration of 0.2 mol/L. The molar ratio MEA/Zn was fixed to 1. Finally, trimethyl borate was added to provide a stock solution with specific molar ratio of boron to zinc 0.2 at.%. The solution was dropped onto corning glass (1737) substrates, which were rotated at 2500 rpm for 30 s. After each deposition, the resulting film was then dried in air at 230°C for 15 min in an oven in order to remove the solvents and organic compounds by evaporation. The boron doped films were annealed in air at 450°C for one hour. The flow chart for the processing of ZnO:B thin films is shown in Fig.1. Crystallinities of the ZnO: B films were confirmed by x-ray diffractometer (PAN alytical X'pert PRO) using CuK$_{á}$ radiation (ë = 1.5140Å). Optical transmittance measurements were carried out using UV-VIS-NIR spectrophotometer (Shimadzu Solid Spec 3700).

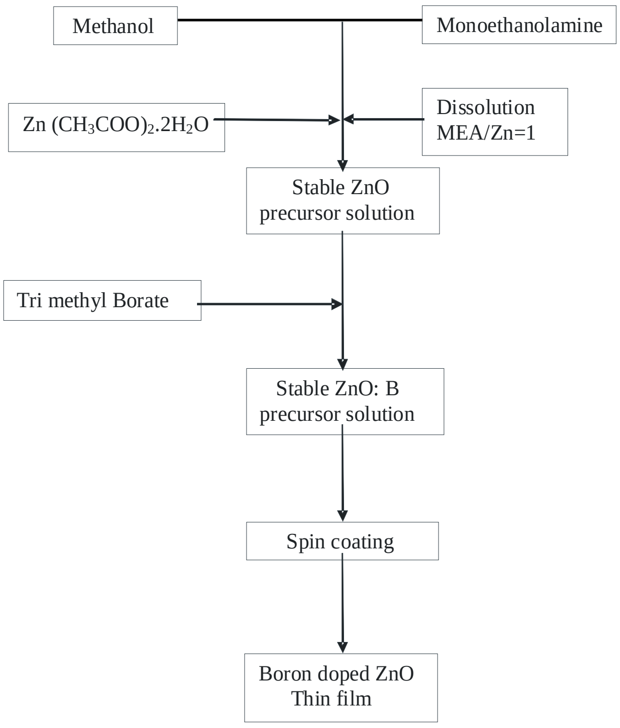

Fig.1 Flow chart for Born doped ZnO films prepared by sol- gel method

RESULTS AND DISCUSSION

Structural Properties

The X-ray diffraction spectra of boron doped ZnO films deposited by sol gel techniques at 0.2 at.% boron doping concentration is shown in Fig. 2. The X-ray spectra of undoped ZnO prepared by sol-gel are also inducted in the figure for the comparison. The registered X-ray spectra fit to a ZnO hexagonal wurtzite structure without any new reflections. No additional peaks of ZnO phases and the traces of either B or Zn metals have been observed. This indicates that the B^{+3} are substituted at Zn^{+2} sites without changing the ZnO structure.

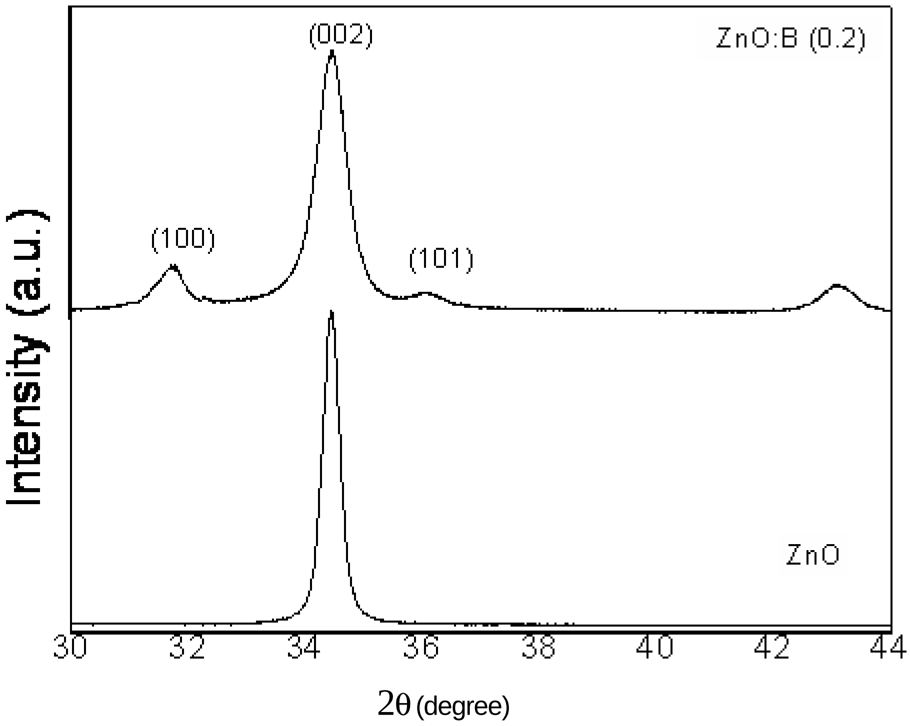

Fig.2. XRD Spectra of undoped and boron doped ZnO films.

Optical Properties

The optical transmission spectra of the film deposited on corning glass substrates taken in the wavelength region 300-1000 nm are shown in Fig 3. The maximum transmittance in visible region was found to be 87%. This value is significantly higher than those of most transparent conductive oxides. The transmission data were used to evaluate absorption coefficients of the boron doped thin films at different wavelengths. At the absorption edge, the absorption coefficient (*á*) is given by the following relation [10]

$$T \quad \exp^{(\quad d)}$$

where *d* is film thickness and á is absorption coefficient.

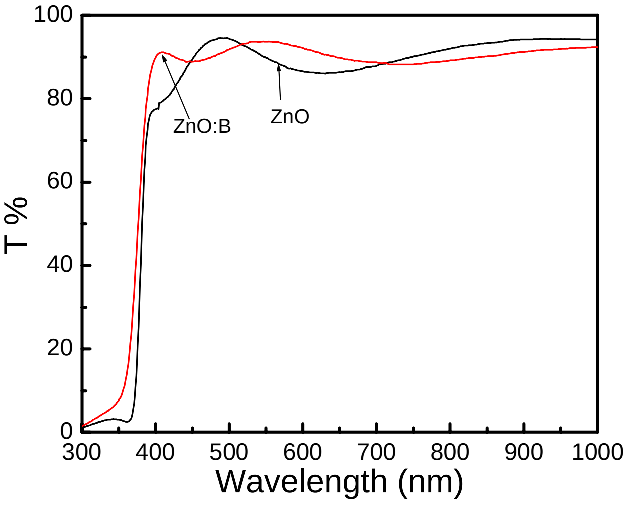

Fig.3. Optical transmittance spectra of undoped and boron doped ZnO films.

In the direct transition semiconductor, the absorption coefficient, obeys the following relationship with optical energy band gap (Eg) [4] $\alpha h\nu = B\left(h\nu - E_g\right)^{1/2}$

where *B* is a constant, *h* is Planck's constant and *v* is the frequency of incident photon. In Fig.4, it is clearly shown that band gap is increased from 3.24 eV to 3.26 eV on boron doping in ZnO. The variation in band gap of films of different thickness could be explained by Burstein–Moss shift [1].

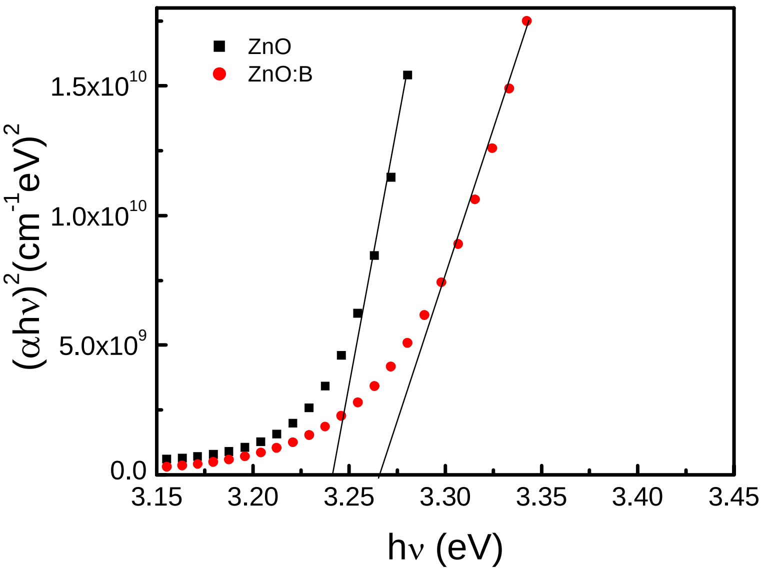

Fig.4. Optical band gap of undoped and boron doped ZnO films.

CONCLUSION

The effect of boron doping on the structural and optical properties of ZnO thin films was studied. The optical transmittance of ZnO: B film is higher than the ZnO film. Band gap of Zinc oxide is increased after boron doping in Zinc Oxide.

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DESIGN AND SIMULATION OF A LOW ACTUATION VOLTAGE WIDEBAND RF MEMS SWITCH ON HFSS

(Received 04.10.2009, Revised 07.12.2009)

***Nitin, **Vipul Sharma, ***S.S. Pattnaik, **Tanuj Garg ***S. Devi and **Sameer Kaul**

Many RF MEMS switch topologies have been reported and they all show superior RF characteristics compared to semiconductors based components. This work investigates the simulation of RF MEMS switch. The switch is a capacitive type, which is actuated by electrostatic force. The structure of the switch consists of a CPW (coplanar waveguide) transmission lines and a suspended membrane. The switch has an insertion loss of less than 0.25dB and an isolation of more than 45dB over the frequency range from 1 to 10GHz. Simulated results show that the pull in voltage of the switch is about 10 volts.

INTRODUCTION

The low insertion loss and high isolation RF characteristics over a wide frequency range make MEMS switches the most attractive devices for reconfigurable antenna and integrated circuit applications [1]. Most of RF MEMS switches utilize rotary [5], cantilever [7], and membrane structures [2-4]. They have demonstrated superior RF characteristics compared to active-device-base solid state switches such as FET and diode switches. These RF MEMS devices, however, require very high actuation voltages (usually 30-50 volts). The high voltage operation is far beyond standard monolithic microwave IC operation, which is around 5 Volts DC biased operation. There is a crucial need for improved apparatus and method which address this drawback of known RF MEMS switches. In this paper we proposed a RF MEMS switch for low voltage operation [6]. The experimental results of the RF MEMS switch show a promising low voltage operation and excellent RF performance over the frequency band from 1GHz to10 GHz. This paper presents the *simulation* and measurement results of a “cantilever” RF MEMS switch. The simulated results show that the device switching voltage is 10 volts. When the device is operated in an ‘on’ state, an insertion loss of less than 0.25 dB at 10 GHz with a return loss of -36 dB at 10 GHz was measured. When operated in its ‘off’ state, an isolation of better than 45 dB over the frequency band from 1 to 10GHz was achieved.

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* Dehradun Institute of Technology, Dehradun
** Faculty of Engineering and Technology, Gurukul Kangri University, Haridwar
*** National Institute Technical Teachers Training and Reaserch, Chandigarh

DEVICE STRUCTURE

The schematic cross-sectional view of the RF MEMS switch is shown in Fig. 1. The RF signals are guided by a coplanar waveguide (CPW) structure, which is composed of a signal line and ground planes on both sides. The CPW in this switch is disconnected in the middle, when on applying the voltage over the membrane, the membrane can be used to join the two disconnected ends of the CPW and hence make the switch ON. The actuation voltage provides an electrostatic force to make the membrane move up and down. When voltage is applied on the membrane, the voltage on the input port of the CPW will directly pass to the output port through the membrane and this forms the “on” condition of the switch. On the other hand when no voltage is applied to the membrane , no electrostatic force is developed between the membrane and the CPW as a result of this, the voltage through the CPW can’t get transferred to the output port and this forms “off” condition of the switch.

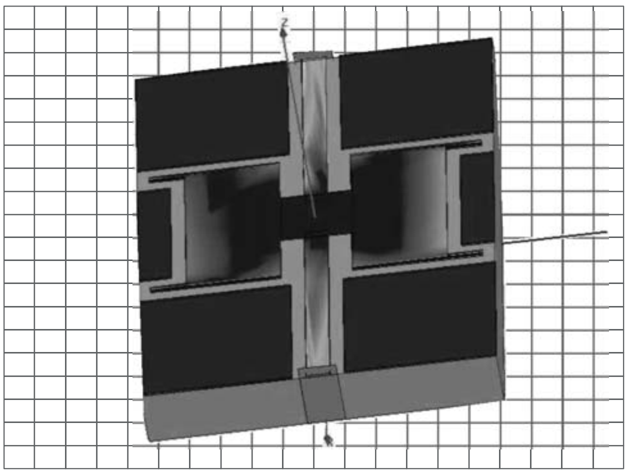


Fig. 1: The schematic cross-sectional side view of RF MEMS switch with hinge configuration

DESIGN PARAMETERS OF THE SWITCH

Coplanar waveguide- Substrate- Rogers RO 4232(tm), permittivity- 3.2, dimensions 420*300 µm^2, height- 50µm.

Metal line- Copper, Dimensions 420*20 µm^2, Height -2.1 µm, Slot -12 µm.

Ground plane- Copper, dimensions- 140*128 µm^2, height-2.1µm

Switch - Central contact- gold, dimensions 56*60 µm^2, height 1 µm

Side cantilevers- Gold, dimensions -150*80 µm^2, height -1 µm

Anchor- Gold, dimension- 4*4 µm^2, height -1 µm.

Joint between cantilever and anchor- Gold, dimensions- 4*30 µm^2, height-1 µm.

Cut in the metal line- 40*20 µm$^{2.}$

Gap between switch and metal line - 1 µm.

Height of Radiation box - 1mm.

Structures of the switch in on and off positions are shown in Fig.2 -3.

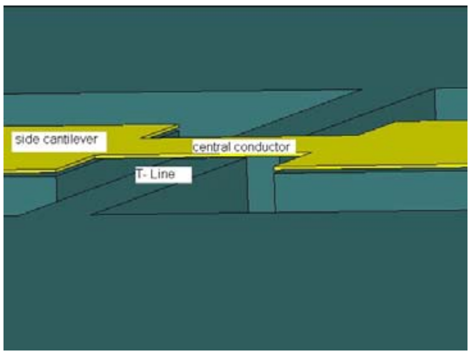


Fig. 2 Switch in OFF state

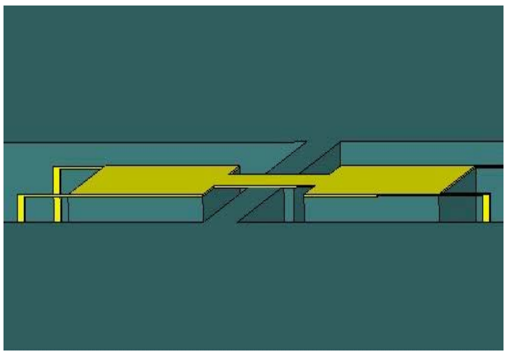
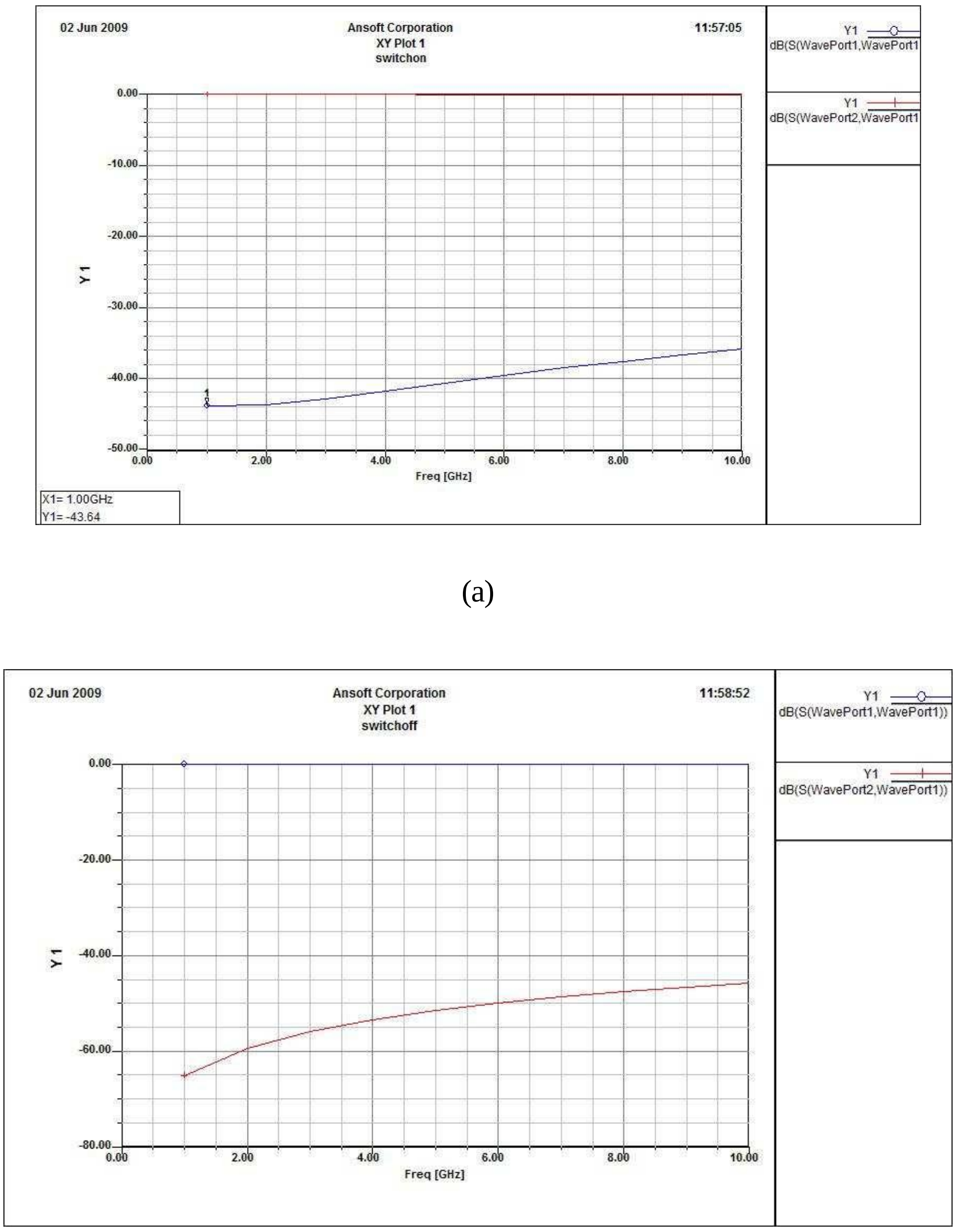


Fig.3 Switch in ON state

RESULTS AND CONCLUSION

In conclusion, structures using a promisingly low actuation voltage of 10 volts for a cantilever switch have been demonstrated. The S-parameters of the device were measured from 1 to 10 **G**Hz using HFSS. In switch 'on' state), an insertion loss of 0.25 dB at 1 GHz and 0.5 dB at 10 GHz with a return loss of - 44 dB at 1GHz and -36 dB at 10 GHz was measured and depicted in Fig. 4 (a). In switch "off" state, an isolation better than 45 dB over the frequency band from 1 to 10 GHz was measured and seen in Fig. 4(b) . The low voltage switches will play an important role for the next generation MEMS based wireless communication system.



(a)

(b)

Fig. 4: The RF performance of MEMS switch for (a) ‘switch on’ state and (b) ‘switch off state.

Acknowledgement: The authors are thankful to DEAL, Dehradun, for the technical and lab support they provided.

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(4) प्रकाशक का नाम : प्रो0 ए. के. चोपड़ा
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व पता : कुलसचिव
गुरुकुल कांगड़ी विश्वविद्यालय
हरिद्वार – 249409
(5) प्रधान सम्पादक : प्रो0 ए. के. चोपड़ा
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हस्ताक्षर
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कुलसचिव

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GRACEFUL GRAPHS FROM HAIRY CYCLES

P. Pradhan[*] and Ajay Kumar[*]

ABSTRACT

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In this paper graceful labelings of subdivision of hairy cycles, subdivision of the pendent edges attached to each vertex of the hariy cycle $C_n \odot 1K_1$ and graceful labeling of two isomorphic copies of the hairy cycle by merging each pendent vertex of one copy hairy cycle to its isomorphic copy have been studied.

Keywords: graceful labeling, hairy cycle, unicyclic graph, corona of graph, subdivision of graphs.

INTRODUCTION

A graph with a graceful labeling is called a graceful graph. The concept of graceful labeling was first introduced by Ringel-Kotzig-Rosa [4], [5], [6] and [7]. By a graceful labeling of a graph *G* with *q* edges, we mean an injection *f:V(G)* {0,1,2..........,q} such that

.Truszcy nski [8] conjectured that all the unicyclic graphs (graphs that have only one cycle) except C_m where *m* 1 or 2 (mod 4), are graceful. A caterpillar is a tree with the property that the removal of all the pendent vertices reduces it to a path. A unicyclic graph G other than a cycle with the property that the removal of any edge from the cycle reduce G to a caterpillar is called hariy cycle. Barrientos [1], [2] proved that all hariy cycles are graceful. A lobstar is a tree with the property that the removal of all pendent vertices reduces it to a caterpillar

$\{1,2,3,\ldots\ldots\ldots\ldots,q\}$

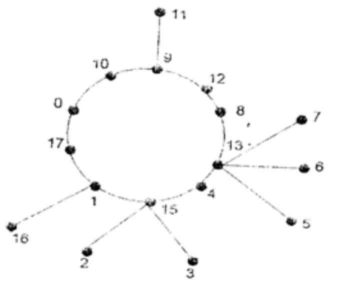

Fig 1: A hairy cycle with a graceful labeling.

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[*] Department of Mathematics and Statistics, Grukula Kangri Vishwavidyalaya, Haridwar, PIN-249404, Uttarakhand, India. E-mail:ppradhan14@gmail.com, ajaygraph.gkv.math@gmail.com

Fruchut and Haray [3] presented a new operation between two graphs, namely, the corona of two graphs. Given two graphs G and H, the corona of G with H, denoted by $G \odot H$ is a graph with

In other words, a corona graph is obtained from graphs G and H by taking one copy of G with order say p, and p copies of H, and then joining the K^{th} vertex of G to every in the K^{th} copy of H by an edge. The corona of and mK_1, i.e $C_n \odot mk_1$ is the example of hairy cycle.

MOTIVATION

Motivated by the results in [1], an algorithm has been developed to construct graceful graphs by attaching pendent edges to each pendent vertex of the hairy c ycle .

Proposed Algroithm: Let G be a graph obtained from the hairy cycle by attaching exactly one pendent edge to every pendent vertex of the hairy cycle . Let

We observe that the distance of every pendent vertex in G from the unicycle is exactly two. Furthermore, the removal of any edge of the unicycle in G reduces it to a lobster. By [1], there exists a gracful labeling f of $C_n \odot 1k_1$ in which a pendent vertex gets the label 0. We know that a hairy cycle is bipartite graph and so G is bipartite. Let $\{A,B\}$ be the bipartition of the vertices of G.

Step I:- If u_1 be the pendent vertex of the hairy cycle at which one pndent edge u_1v_1 E(G) is attached and let $f(u_1) = 0$, $f(v_1) =$ m and $f(v_2) = m-1$. If more than one pendent edges are attached to each pendent vertices of hairy cycle, then $f(u_1) = 0$, $f(v_1) =$ m-1 and $f(v_2) =$ m.

Step II:- If exactly one pendent edge is attached to one pendent vetex of hairy cycle, then $f(u_2)$- 1 otherwise $f(u_2) = 2$.

Step III:- Give $f(v_3) =$ m - 2 for the above two cases and $f(u_3) =1$ for the graph G obtained by attaching two or more pendent edges to every pendent vertices of hairy cycle.

Step IV:- As soon as the two pendent edges of hariy cycle get lables differently, then give the lable to the vetex u_i adjacent to v_1 (when only one pendent edge is attached to the pendent vertex of hairy cycle) or the vertex v_2 (when more than one pendent edges attached to the pendent vertices of hairy cycle) such that u_iv_1 $E(G)$ of u_iv_2 $E(G)$ get the label one less than edge lable u_2v_3 $E(G)$

Step V:- Give the label to the vertex v_4 such that the edge u_2v_4 $E(G)$ gets the ladel one less than the edge of u_iv_1 $E(G)$ or u_iv_2 $E(G)$.

Step VI:- Increase the label of u's and decrease the label of *v*'s until the differently all the edges and vetices of *G* get the distinct label and after that when all the edges and vertices have distinct label then terminate the algorithm.

Theoerm1: Subdivision of cycle in all graceful hairy cycles are graceful hairy cycles.

Proof : It is obvious that by subdivision of cycle in hairy remains a hairy cycle. So, by subdivision of cycle is graceful hairy cycle is graceful hairy cycle.

Theorem 2: Subdivision of the pendent edges attached to each vertex of graceful hairy cycle is a graceful graph.

Proof : The figure 9 shown below proves the theorem.

Theorem3 : Graph obtained from two isomorphic copies of to its isomorphic copy of the second copy of .

Proof : The figure 10 proves the theorem.

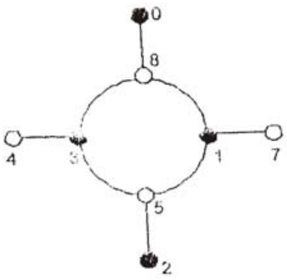


Fig. 2: Hairy Cycle

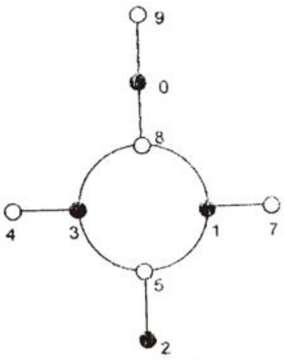


Fig. 3: Hairy Cycle with one pendent edge attached

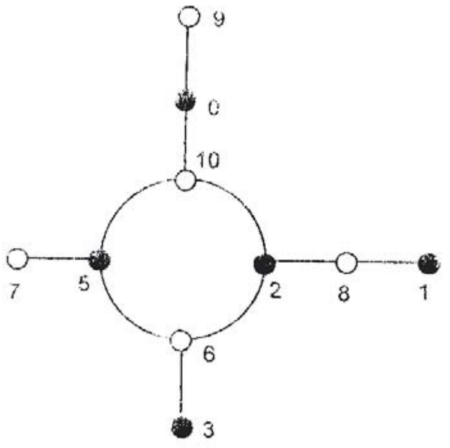


Fig. 4: Hairy Cycle with two pendent edges attached

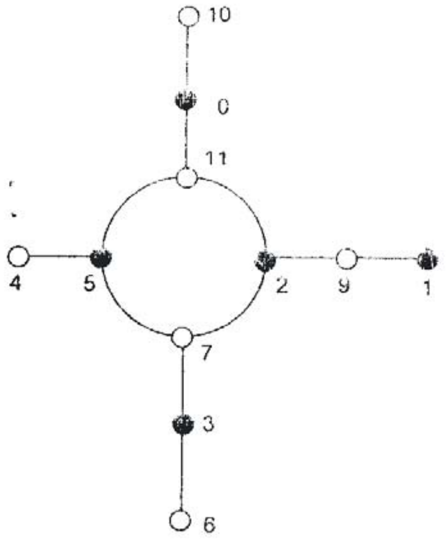


Fig. 5: Hairy Cycle with three pendent edges attached

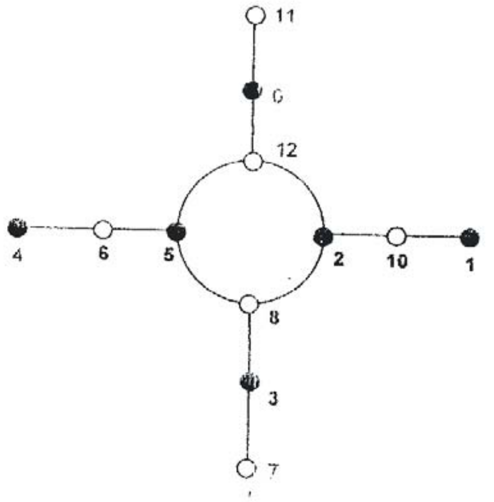


Fig. 6: Hairy Cycle with four pendent edges attached

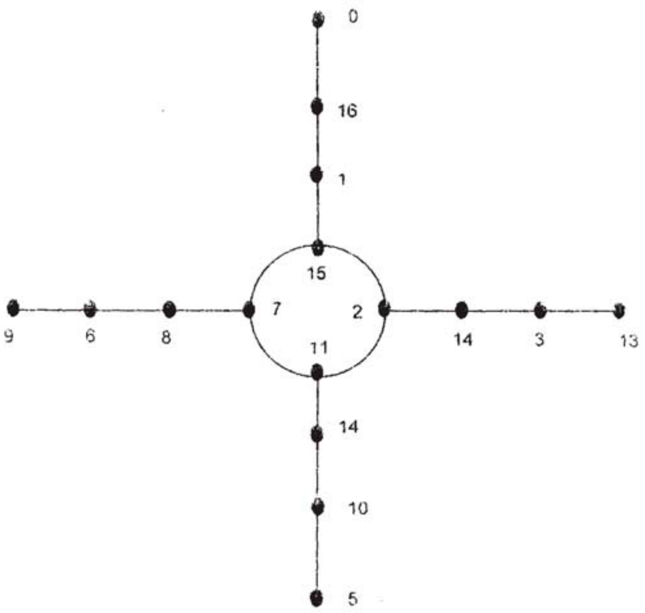
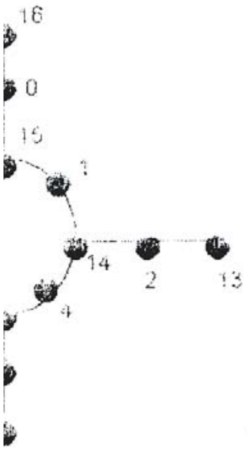


Fig 9 : Subdivision of pendent edges attached to each vertex of hairy cycle



cycle
obtained by
n graceful hairy cycle

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COMMON FIXED POINT OF A PAIR OF QUASI-NONEXPANSIVE AND ASYMPTOTICALLY NONEXPANSIVE MAPPINGS IN CAT(0)-SPACES

José Luiz Corrêa Camargo*

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ABSTRACT

We obtain a fixed point theorem for a quasi-nonexpansive mapping and an asymptotically nonexpansive mapping in the framework of a complete CAT(0) spaces .

Keywords: Quasi-nonexpansive nonexpansive mapping; asymptotically nonexpansive mapping; fixed point.

MSC (2000) 47H10, 54H25

INTRODUCTION

In 1972, K. Goebel and W.A.Kirk introduced a new class of mappings namely asymptotically nonexpansive, which have been studied mostly in the context of uniformly convex normed spaces. On the other hand, in 1967 Diaz and Metcalf [2] introduced the concept of quasi-nonexpansive mappings. Both classes of mappings have been studied intensively under various settings. Our contribuition here is to establish a new result about existence of a common fixed point between two mappings S and T, where S is quasi-nonexpansive and T is an asymptotically expansive mapping, in the context of CAT(0) spaces.

We remark that in 1972, Goebel and Kirk proved the following generalization of the famous Browder-Goehed-Kirk fixed point theorem for nonexpansive mappings.

Proposition 1.1. (see [3]) Nonempty closed convex and bounded subsets of uniformly convex Banach spaces have the FPP for asymptotically nonexpansive mappings.

In 2004, W.A. Kirk obtained a similar result for a CAT(0) space.

Proposition 1.2 (see [6]) Nonempty closed convex and bounded subsets of complete CAT(0) spaces have the fixed point property (FPP) for asymptotically nonexpansive mappings.

Proposition 1.3 (see [4]) Let X be a convex subset of a uniformly convex metric space and $S: X\rightarrow X$ a quasi-nonexpansive mapping whose fixed point set is nonempty. Then

Fix (S) is closed convex.

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*Department of Mathematics Instituto Tecnológico de Aeronáutica (ITA) S.J. dos Campos, SP, Brasil
jlccamargo@yahoo.com.br

At this point for the sake of completeness, we recall some basic definitions and introduce some notations which will be used in this note.

Notation 1.4. If X is a nonempty and $S{:}X \to X$, we denote by Fix (S) the collection of all $p \in X$, such that $S\,(p) = p$

Remark. 1.5 (see [6], [1]). A CAT(0)-space is a specific type of metric space (see [1] for a detailed treatment).

Definition 1.6 (see [5]). Let (M,d) be a metric space. A mapping $T : M \to M$ is said to be asymptotically nonexpansive if there exists a sequence of positive numbers $\{k_n\}$, with $\lim k_n = 1$, when $n \to$, such that

$$d\,(T_n(x), T_n(y)) \leq k_n d\,(x, y) \text{ for any } x, y \in M, \text{ and } n = 1, 2, ...$$

Definition 1.7 (see [4]). Let (X, d) be a metric space. A mapping $f{:}\, X \to X$ is said to be quasi-nonexpansive if $d\,(f\,(x), p) \leq d\,(x,p)$ for all $x \in X$ and $p \in$ Fix (f).

Definition 1.8 (see [4]). Let (X, d) be a metric space and $I = [0,1]$. A mapping $W{:}X{\times}X{\times}I \to X$ is said to be a convex structure on X if for each $(x, y, \lambda) \in X{\times}X{\times}I$ and $z \in X$, $d(z, W(x, y, \lambda) \leq \lambda\, d\,(z,x) + (1\text{-}\lambda)\, d\,(z, y)$.

Definition 1.9 (see [4]). A metric space (X, d) together with a convex structure W is called a convex metric space, which will be denoted by (X, d, W).

Definition 1.10 (see [8]). A convex metric space (X, d, W) is said to be uniformly convex if for any $\varepsilon > 0$, there exists $\mathrm{S} = \mathrm{S}\,(\varepsilon) > 0$ such that for all $r > 0$ and $x, y, z \in X$ with $d\,(z, x) \leq r$, $d\,(z,y) \geq r$ and $d\,(x,y) \geq r\varepsilon$, $d\,(z, W\,(x, y, ½)) \leq r\,(I - S)$.

Remark 1.11 (see [4]). It is easy to conclude that

a) uniformly convex Banach spaces are uniformly convex metric spaces.

b) CAT(0) spaces are also uniformly convex.

Definition 1.12 (see [8]}. Let S be a subset of a convex metric space. S is said be convex if for every $x \in S$, $y \in S$ and $\lambda \in [0,1]$ we have $W\,(x, y, \lambda) \in S$.

THE MAIN RESULT

The main result of our work goes as follows

Theorem 2.1. Let X be a convex and bounded subset of a complete CAT(0)-space. Let $S{:}X \to X$ a quasi-nonexpansive mapping whose fixed point set is nonempty. Suppose that

$T{:}X\to X$ is an asymptotically nonexpansive mapping. If one of the following conditions holds,

a) $SoT = ToS$ (2,1)

or

b) $\|u-T(u)\| \leq N(\|u-S(u)\|,\| T(u) - (SoT)(u) \|)$ (2,2)

for every $u \in X$, where N: $R^{+}\times R^{+}\to R^{+}$ is a function such that

$\mathrm{N}(0, t) < t$, if $t > 0$

or

c) $\| (SoT)(u) - T(u) \| \leq M (\|(\mathrm{T}oS)(u)-T(u)\|)$ (2,3)

for each $u \in X$, where M: $R^{+}\to R^{+}$ is a function such that $M (s) < s$, if $s > 0$; then Fix(S)   Fix (T) $\neq$ Ø, that is, S and T have a common fixed point in X.

Proof: From [6], we observe that the metric space CAT(0) is uniformly convex. Further, Fix (S) X and X is supposed bounded. So, Fix(S) is also bounded and, by Proposition 1.3, Fix(S) is nonempty closed and convex subset of X.

If $SoT = ToS$, it is easy to see that T: Fix (S) $\to$ Fix(S), because in this case $u \in$ Fix (S) implies $u \in X$ and $S (u) = u$ and therefore $TS (u) = ST(u) =.T(u)$. If S and T satisfies (2.2.) or (2.3.), and u Fix(S) and $T(u)$ is not in Fix(S), we obtain imediately a contradiction, since we have $N(0, t) < t$, if $t > 0$ and $M(s) < s$, if $s > 0$. Hence under the hypothesis (2.2) or (2.3), we get T(Fix(S)) $\in$ Fix (S). Therefore, since T: Fix(s) $\to$ Fix(S) also is asymptotically nonexpansive mapping, we note that all considerations for appliying Proposition 1.2 are fullfilled.

$\cap$ Hence, there exists at least a $p\in$ Fix(S) such that $T(p) = p$, that is, Fix (S)   Fix(T) $\neq$ Ø. This completes the proof.

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SEXUAL VARIATIONS IN BLOOD CONSTITUENTS OF FRESHWATER CATFISH, *CLARIAS BATRACHUS*

Sudhish Chandra* and Praveen Kumar**

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ABSTRACT

Sex related variations in twelve blood constituents have been observed in freshwater catfish *Clarias batrachus*, whose population is dwindling at alarming level in and around Lucknow. Interestingly most of the parameters like blood urea, ascorbic acid, total serum proteins, cholesterol, acid and alkaline phosphatase, amylase, SGOT, SOPT and LDH revealed higher levels in females than their respective males, except haemoglobin and serum iron levels. The maximal difference of 33.88% was noticed in serum acid phosphatase while minimal 6.06% in haemoglobin levels between the two sexes. Sex related trends in biochemical blood constituents which may help to understand physiological status offish during breeding, culture and propagation practices have been discussed, comparing to some of those reported for other fishes.

INTRODUCTION

Biochemical characteristics of blood are among the important indices of the status of internal en vi ronment of fish organism [6]. Various body and blood consti tuents of fishes are known to differ in two sexes, since they are related to maturity, breeding cycles and varying growth rate of fish. Hacmatology no doubt plays an important role in the diagnosis of disease in fish and also in the assessment of the effects of pollution in fish life, unless basic work is done to establish normal haematological ranges this will be difficult [9, 21]. Sex ratio varies considerably from species to species, it differs from one population to another of the same species and may vary from year to year in the same population [26]. Sex related variations in blood constituents have been observed [3 - 8. 11, 13. 14. 20. 31] under specific and varied ecophysiological conditions in different fishes. Following accidental escape of some exotic fishes in Indian streams, population of many native fishes are dwindling at alarming level, Therefore, fishery management should ens me that population of these fishes are sustained at commercially viable level, while meeting societal and economic need.

Clarias batrachus, a prominent catfish is highly popular in India as expensive tablefish, due to its higher nutritional quality as it contains high value of physiological available iron and copper, essential for haemoglobin synthesis and easy digestibility because of low fat content, is facing tough competition in its population due to introduction of Thai mangur *Clarias gariepinus* in its natural habitat [34]. Present paper embodies observations on twelve biochemical blood constituents of freshwater catfish *C. batrachus* in the two sexes, living under natural conditions, which may help to understand physiological status of fish during breeding, culture and propagation

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*P.G. Department of Zoology, B.S.N.V. College, Lucknow-226001
**Department of Zoology, D.A.V. P.G. College, Lucknow-226004

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STOCHASTIC ANALYSIS OF GOMPERTZ GOWRTH WITH PERIODICALLY VARYING GROWTH AND DEVELOPMENT PARAMETERS

N.G. Sarkar*

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ABSTRACT

We present a stochastic analysis of a population obeying Gompertz growth equation with periodic growth coeffcient and development parameter under the influence of randomly fluctuating environment.

Keywords: Gompertz Equation, Periodic Growth Coefficien, Periodic Development Parameter, Fluctuation.

INTRODUCTION

The growth is an important significant part of life processes [2,9]. Gompertz [4] model equation which is generalization of Malthusian [6] growth model has found wide application in different growth processes of men, animals, plants, tumour etc. [5]. Gompertz quardratic equation plays a significant role in the modelling of plants growth. It is of vital importance in agriculture and forestry [8]. Gompertz model equation for single-species population is given by [1]

$$\frac{dN(t)}{dt} \quad a_0\, e^{\ \ kt} N(t) \tag{1.1}$$

where $N(t)$ is the population size at any time t, a_0 is the growth coefficient, k is the development or senescence parameter. In the above model equation both the parameters $a_0$and k are constants. A great phenomena are cyclic or oscillatory in nature. Due to diurnal or annual changes, there are numerous environmental factors like themperature, sunlight, soil or moisture, nutrients etc. cause annual cyclic variations of the growth and development parameters of trees [1,15]. Besides, the randomly fluctuating environment also influences the growth process [10-14,16,17]. In recent years interest on the mathematical modelling of biological and tumor growths based on Gompertz equation and its present paper is to study the overall effects of the periodically varying growth coefficient and the randomly fluctuating environment of the growth behaviours of a population obeying Gompertz model equation. The equation (1.1) can not be analysed directly by adding fluctuating term in a_0 , k and in the whole system. So we slightly change this model by solving $N(0) = N_0$ at $t = 0$ then by simplification we get the changed model as follows :

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*Department of Mathematics, Kalyani Govt. Engineering College Kalyani-741235, Nadia, West Bengal, India

$$\frac{d\ln(N(t)/N_0)}{dt} \qquad k\ln(N(t)/N_0) \quad a_0$$

or

$$\frac{dy(t)}{dt} \qquad k\,y(t) \quad a_0 \tag{1.2}$$

where

$$y(t) \quad \ln N(t) \quad \ln N_0 \tag{1.3}$$

y (*t*) is increasing function as *N* (*t*) is increasing function.

GOMPERTZ GROWTH WITH SINUSOIDALLY VARYING GROWTH COEFFICIENT IN RANDOM ENVIRONMENT

Now we shall start with a simple harmonic growth coefficient

$$a_0(t) \qquad {}_0 \qquad \sin \quad t; \qquad {}_0, \qquad 0 \tag{2.1}$$

This implies that the time dependent growth coefficient (*t*) has average value a_0 and oscillation with amplitude about this average value at frequency = 2π/*T*, *T* is theperiod of oscillation. The oscillation increases as increases and no oscillation for = 0. For small oscillation in and satisfy the condition:

(2.2)

We have considered here *k* is constant. To take account of the influence of the randomly fluctuating environment on the growth parameter a_0 (*t*), we modify equation (2.1) by adding a random perturabation or force (*t*) satisfying the conditions [3].

(2.3)

The sign < > denotes the average over the ensemble of stochastic forces. Then (2.1) becomes

$$a_0(t) \qquad {}_0 \qquad \sin \quad t \qquad (t). \tag{2.4}$$

Putting equation (2.4) in (1.2) we get

(2.5)

which is a stochastic differential equation. According to Gardiner [3] equation (2.5) is the form of

$$\frac{dy(t)}{dt} \quad \{a(t) \quad b(t)\,y(t)\} \quad \{h(t) \quad g(t)\,y(t)\} \quad (t)$$

where *a* (*t*) = $_0$ + sin , *b* (*t*) = - *k*, *h* (*t*) = 1, *g* (*t*) = 0 and properties of are given by equation (2, 3), we use for *n* th order moment equation

$$\frac{d \quad y^n(t)}{dt} \quad y^n(t) \quad \left[n\,b(t) \quad \frac{n(n \quad 1)}{2} g^2(t) \right]$$

$$y^{n \quad 1}(t) \quad \left[n\,a(t) \quad n(n \quad 1) h(t(g(t))) \right]$$

$$y^{n \quad 2}(t) \quad \left[\frac{n(n \quad 1)}{2} h^2(t) \right] \tag{2.6}$$

Taking n = 1 in equation (2.6) the mean < *y* (*t*) is then given by

$$\frac{d \quad y(t) \quad _1}{dt} \quad k \quad y(t) \quad _1 \quad _0 \quad \sin \quad t \tag{2.7}$$

The solution of the equation (2.7) is given by

$$y(t) \quad _1 \quad \left(\quad y(0) \quad _1 \quad \frac{_0}{k} \quad \frac{}{k^2 \quad ^2} \right) e^{kt} \quad \frac{_0}{k}$$

$$\frac{(k \sin \quad t \qquad \cos \quad t)}{k^2 \quad ^2} \tag{2.8}$$

$y(0)$ $_1$

where is the initial value of $y(t)$ $_1$. So that $y(t)$ $_1$ is exponentially and periodically increases with time t but for large t, $y(t)$ $_1$ tends to be a finite value with oscillation. For the graph of the equation (2.8) we shall take $_0$ = 0.280, = 0.520, *k* = 0.0747, = 0.524, *T* = 12 months, = 1.5. The graph of equation (2.8) for average $y(t)$ $_1$ has been shown time (*t*) versus $y(t)$ $_1$.

The equation of second order moment $y^2(t)$ $_1$ is given by putting *n* = 2 in equation (2.6), then after calculation

$$\frac{d\quad y^2(t)\ _1}{dt}\quad 2k\quad y^2(t)\ _1\quad 2(\alpha_0\quad \alpha\sin\omega t)$$

$$\left\{Ae^{\ kt}\quad \frac{\alpha_0}{k}\quad \frac{\alpha(k\sin\omega t\quad \omega\cos\omega t)}{k^2\quad \omega^2}\right\} \tag{2.9}$$

where

$$A\qquad y(0)\ _1\quad \frac{\ _0}{k}\quad \frac{}{k^2\qquad ^2}$$

with intial condition at t = 0. The complete solution of equation (2.9) is

$$y^2(t)\ _1\quad \Bigg[\quad y^2(0)\ _1\quad \left\{\frac{2A\ _0}{k}\quad \frac{2A}{k^2\qquad ^2}\quad \frac{\ _0^2}{k^2}\quad \frac{6\ _0\ k}{(4k^2\qquad ^2)}\right.$$

$$\left.\frac{2\ _0}{k(4k^2\qquad ^2)}\quad \frac{^2}{2(k^2\qquad ^2)}\quad \frac{^2\ k}{2(k^2\qquad ^2)^2}\quad \frac{1}{2k^2}\right\}\Bigg]e^{\ 2kt}$$

$$2A\left[\frac{\ _0}{A}\quad \frac{}{k^2\qquad ^2}(k\sin\quad t\qquad \cos\quad t)\right]e^{2kt}$$

$$\left[\frac{\ _0^2}{k^2}\quad \frac{2\ _0\quad \sin\quad t}{4k^2\qquad ^2}\quad \frac{6\ _0\ k\quad \cos\quad t}{(k^2\qquad ^2)(4k^2\qquad ^2)}\right.$$

$$\frac{2\ _0\ (2k\sin\quad t\qquad \cos\quad t)}{k(4k^2\qquad ^2)}\quad \frac{^2}{2(k^2\qquad ^2)}\quad \frac{^2\ ^2k}{2(k^2\qquad ^2)^2}$$

$$\left.\frac{^2\ ^2}{2(k^2\qquad ^2)}\quad \frac{1}{2k}\right] \tag{2.10}$$

where

$$A\qquad y(0)\ _1\quad \frac{\ _0}{k}\quad \frac{}{k^2\qquad ^2}$$

Fluctuation is exponentially and periodically increasing with time t, and remains periodically finite for large *t*. For the graph of (2.10)we shall take $_0$ = 0.280, = 0.520, k = 0.774, =0.524, T = 12 months, = 1.5, $y^2(t)$ $_1$ = 1.2 then A_1 = -1.2757.

The graph of equation (2.10) for variance $y^2(t)$ $_1$ has been shown time (t) versus $y^2(t)$ $_1$.

GOMPERTZIAN GROWTH WITH PERIODICALLY FLUCTUATING DEVELOPMENT PARAMETER IN RANDOM ENVIRONMENT

Let us now suppose that the development parameter, *k*, varies periodically with time *t* because of the diurnal or annual changes of environment. If *T* be the period of ocillation in developmental parameter, then the frequency of oscillation is defined by 1/*T* and angular frequency is defined by = 2 / *T*. The simplest possible example of oscillating with angular frequency is obtained by assuming for the development parameter, *k* [7],

(3.1)

where

$$_0, \quad 0 \quad and\, 0 \qquad 1 \tag{3.2}$$

Here $_0$ is the average value of *k* over a long period of time interval and the condition (3.2) implies the small fluctuation in *k* relative to $_0$. To take account of the influence of randomly fluctuating environment on the development parameter, we add in (3.1) a small perturbing term $\varepsilon(t)$ where properties are given by equation (2.3), then

$$k \quad k(t) \qquad _0 \qquad \cos \quad t \qquad (t) \tag{3.3}$$

Putting (3.3) in (1.2) we get

$$\frac{dy(t)}{dt} \quad \alpha_0 \quad (\beta_0 \quad \beta\cos\omega t)y(t) \quad (\quad y(t))\varepsilon(t)$$

$k(t)(0)$ $_0$

$$\cos \quad t \qquad \left[a(t) \quad b(t)y(t)\right] \quad \left[h(t) \quad g(t)y(t)\right]\varepsilon(t) \tag{3.4}$$

$$a(t) \qquad _0, b(t) \qquad (\quad _0 \qquad \cos \quad t), h(t) \quad 0, g(t) \quad 1.$$

Now using the *n*th order moment equation (2.6) and putting *n* = 1 for mean $y(t)$ $_2$, we get

$$\frac{d \quad y(t) \quad _2}{dt} \quad (\quad _0 \qquad \cos \quad t) \quad y(t) \quad _2 \qquad _0 \tag{3.5}$$

The solution of the equation (3.5) is

$$A_2 \qquad y(t) \quad _2 \quad A_2 e^{\;_0 - \sin \quad t} \qquad a_0 \left\{ \frac{1}{\quad _0} \qquad \frac{(\quad _0 \sin \quad t \qquad \cos \quad t)}{(\quad _0^2 \qquad ^2)} \right\} e^{- \sin \quad t} \tag{3.6}$$

where

$$A_2 \quad y(0)_{2} \quad \alpha_0\left(\frac{1}{\beta_0} \quad \frac{\beta\omega}{\beta_1^2 \quad \omega^2}\right) \tag{3.6a}$$

and $y(0)_{2}$ is the intial value of $y(t)_{2}$. So that $y(t)_{2}$ is exponetially and periodically increasing with time *t*, $y(t)_{2}$ tends to be a periodically constant value. For graph we shall take $_0 = 0.4$, $_0 = 0.65$, $= 0.02$, $= 0.524$, $T = 12$ months, $= 1.5$. The graph of equation (3.6) for average $y(t)_{2}$ has been shown time (*t*) versus $y(t)_{2}$.

The equation of the second order moment $y^2(t)_{2}$ is given y putting $n = 2$ in equation (2.6), then

$$\frac{d \quad y^2(t)_{2}}{dt} \quad y^2(t)_{2}\left[\ 2(\ _0 \quad \cos\ \ t) \quad 1\right] \quad 2\ _0 \quad y(t)_{2}$$

Using the equation (3.6) in the above equation we get

$$\frac{d \quad y^2(t)_{2}}{dt} \quad \{2(\ _0 \quad \cos\ \ t) \quad 1\} \quad y^2(t)_{2}$$

$$2a_0\left[A_2 e^{\beta_0 t \quad \frac{\beta}{\omega}\sin\omega t} \quad a_0\left\{\frac{1}{\beta_0^2} \quad \frac{\beta(\beta_0 \sin\omega t \quad \omega\cos\omega t)}{\omega(\beta_0^2 \quad \omega^2)}\right\}e^{\frac{\beta}{\omega}\sin\omega t}\right] \tag{3.7}$$

Final solution of the equation (3.7) for the moment $y^2(t)_{2}$ is given by

$$\begin{aligned}
y^2(t) \quad _2 \; \Big[\; y^2(0) \quad _2 \quad & \frac{2a_0A_2}{\beta_0 \quad 1} \quad \frac{2a_0A_2\beta}{(\beta_0 \quad 1)^2 \quad \omega^2} \quad \frac{2a_0^2}{\beta_0} \quad \frac{2a_0^2\beta^2\beta_0}{2\omega^2(\beta_0^2 \quad \omega^2)(2\beta_0 \quad 1} \\
& \left(\frac{2a_0^2\beta^2}{\beta_0^2 \quad \omega^2} \quad \frac{2a_0^2\beta}{\beta_0}\right)\frac{1}{(2\beta_0 \quad 1)^2 \quad \omega^2} \quad \frac{2a_0^2\beta(2\beta_0 \quad 1)}{(\beta_0^2 \quad \omega^2)(2\beta_0 \quad 1)^2 \quad \omega^2} \\
& \frac{2a_0^2\beta_0\beta^2(2\beta_0 \quad 1)}{2\omega^2(\beta_0^2 \quad \omega^2)(2\beta_0 \quad 1)^2 \quad 4\omega^2}\Bigg] \; e^{(2\beta_0 \quad 1)t \quad \frac{2\beta}{\omega}\sin\omega t} \\
& \left(\frac{2a_0^2}{\beta_0} \quad \frac{2a_0^2\beta^2\beta_0}{2\omega^2(\beta_0^2 \quad \omega^2)(2\beta_0 \quad 1)}\right) \left(\frac{2a_0\beta\beta_0}{\omega(\beta_0^2 \quad \omega^2)} \quad \frac{2a_0^2\beta}{\beta_0\omega}\right) \\
& \Bigg[\frac{((2\beta_0 \quad 1)\sin\omega t \quad \cos\omega t)}{(2\beta_0 \quad 1)^2 \quad ^2} \quad \frac{2a_0^2\beta((2\beta_0 \quad 1)\cos\omega t \quad \omega\sin\omega t)}{(\beta_0^2 \quad \omega^2)((2\beta_0 \quad 1)^2 \quad \omega^2)} \\
& \frac{2a_0^2\beta^2\beta_0((2\beta_0 \quad 1)\cos\omega t \quad 2\omega\sin\omega t)}{2\omega^2(\beta_0^2 \quad \omega^2)((2\beta_0 \quad 1)^2 \quad 4\omega^2)} \qquad (3.8) \\
& \frac{2a_0^2\beta^2((2\beta_0 \quad 1)\sin 2\omega t \quad 2\omega\cos 2\omega t}{2\omega(\beta_0^2 \quad \omega^2)((2\beta_0 \quad 1)^2 \quad 4\omega^2)}\Bigg] \; e^{\frac{2\beta}{\omega}\sin\omega t}
\end{aligned}$$

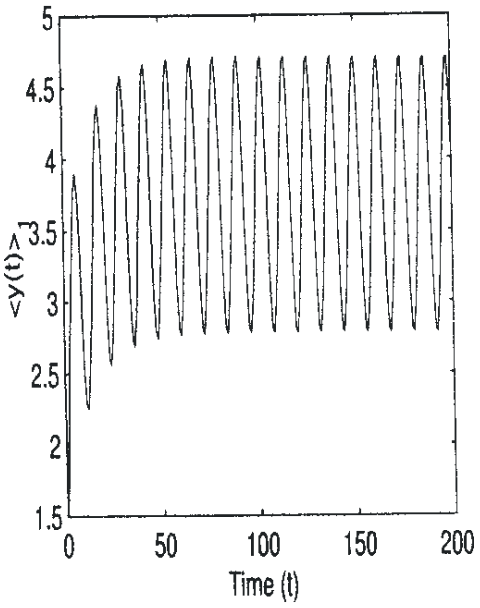
where A_2 is given by (3.6a).

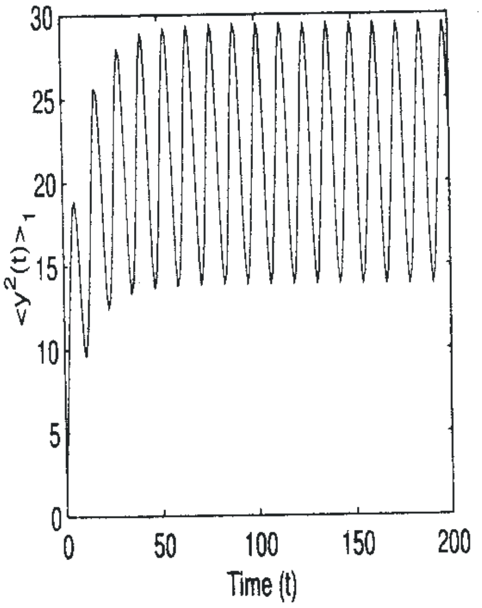
It is clear that for periodically finite fluctuation when t , the required condition

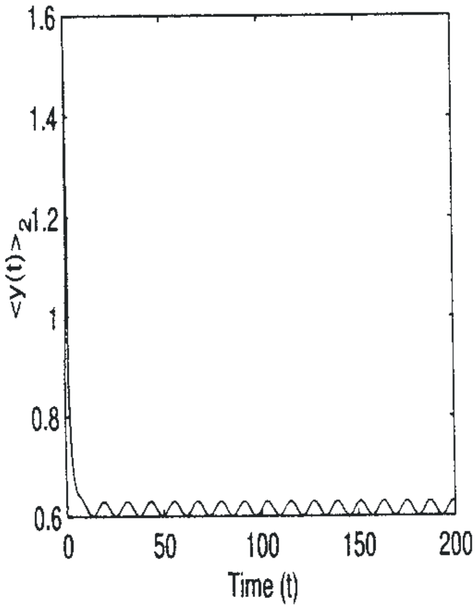
$y(0)_0 \frac{1}{2} \; ^2$ is . When $_0 \; \frac{1}{2}$, the fluction is periodically infinite for large t. For the graph we shall take a_0 = 0.4, $_0$ = 0.65, = 0.02, = 0.524, T = 12 months, = 1.5, $y^2(0) \; _2$ = 1.2, then A_2 = 0.8906, $_0$-1 = 0.35, 2 $_0$ - 1 = 0.3. The graph of equation (3.8) for variance $y^2(t) \; _2$ has been shown time (t) versus $y^2(t) \; _2$.

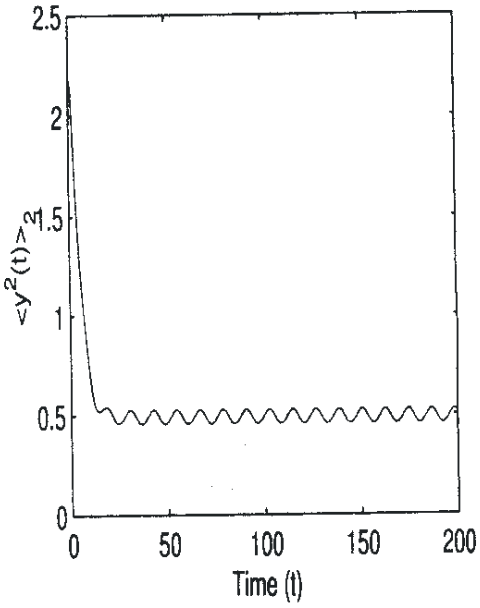
STOCHASTIC RESULTS AND BIOLOGICAL SIGNIFICANCES

Many of the growth phenomena are cyclic or oscillatory in nature. For example, a small tree is planted, begins its growth and total mass-roots, trunk and limbs and leaves continually increases. During the numerous years the tree is growing, its total mass during a year


5
4.5
4
3.5
3
2.5
2
1.5
$<y(t)>_1$
0
50
100
150
200
Time (t)


30
25
20
15
10
5
0
$<y^2(t)>_1$
0
50
100
150
200
Time (t)


1.6
1.4
1.2
1
0.8
0.6
$<y(t)>_2$
0
50
100
150
200
Time (t)


2.5
2
1.5
1
0.5
0
$<y^2(t)>_2$
0
50
100
150
200
Time (t)

oscillates about an average value: leaves and possibly fruits appear in the storing, the tree is in full foliage during the summer, leaves are shed in the autumn and the basic growth pattern, oscillates in nature which is superimposed on the primary average growth of the tree. Clearly, numerous environmental factors-temperature, sunlight, soil, moistrue, nutrient and like-cause this annual cycle of variations in the growth of the tree [1].

In the present paper we have studied the effects of two factors on the growth of plants for example, a tree. These are the effects of cyclic or periodic variation of environmental parameters (such as growth coefficient and development parameter) and the effect of the randomly fluctuating environment. The actual growth of a tree during a year depends on time-ranging ambient conditions of temperature and rainfall which changes from month to month but are essentially repeated a year later. Then primary productivity of a body of water, its biomass and oxygen generation by photosynthesis, are time-dependent due to diurnal varition of light intensity.

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DAMPED VIBRATIONS OF NON HOMOGENOUS CIRCULAR PLATE OF PARABOLICALLY VARYING THICKNESS RESTING ON ELASTIC FOUNDATION

Manu Gupta* & Robin**

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ABSTRACT

The research in the field of vibration is unceasingly acquiring immense importance in modern science as it has great significance in every field of applied science today. The effect of vibration on design of machine, multistoried building, radio-telescope, nuclear reactor technology, earthquake resistant structure and various other engineering structures is of immense importance. Today we are going ahead in space technology where various problems of plates continuously supported by elastic and visco-elastic media are considered. Also effect of damping is applied for practical use.

In this paper Damped vibrations of Non homogenous circular plate of parabolically varying thickness resting on elastic foundation have been studied. The fourth order differential equation of motion is solved by the method of Frobenius. Here the transverse displacement equation of motion is solved by the method of Frobenius. Here the transverse displacement is expressed as an infinite series in terms of radial coordinates. The frequencies, deflection functions and moment parameters corresponding to the first two modes of vibrations are computed for the circular plate with clamped & simply supported edge conditions for various values of taper constants and damping parameters viz. Non homogeneity, Damping, elastic foundation.

Keywords: Non homogeneity, damping, elastic foundation, taper constant

INTRODUCTION

A mathematical model of Damped vibrations of Non homogenous circular plate of parabolically varying thickness resting on elastic foundation have been studied on the basis of classical plate theory. The fourth order differential equation of motion is solved by the method of Frobenius.

Here, again the transverse displacement is expressed as an infinite series in terms of radial coordinates. The frequencies, deflection functions and moment parameters corresponding

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* Department of Mathematics, J.V. Jain (P.G.) College, Saharanpur (Uttar Pradeash) India.
Email Address - gupta_manu13@rediffmail.com
** Now at Department of Mathematics, Roorkee College of Pharmacy University, Roorkee(Uttarakhand) India.Email Address - robin.1986@rediffmail.com

to the first two modes of vibrations are computed for the circular plate with clamped & simply supported edge conditions for various values of taper constants and damping parameters viz. Non homogeneity, Damping, elastic foundation.

EQUATION OF MOTION

The governing differential equation of the transverse motion of a Non homogenous plate of variable thickness under damping and elastic foundation given as

$$^2(D\ \ ^2w)\quad (1\quad v)(\frac{^2D\ \ ^2w}{y^2\quad x^2}\quad \frac{^2D\ \ ^2w}{x\ y\ \ x\ y}\quad \frac{^2D\ \ ^2w}{x^2\quad y^2})\quad h\frac{^2w}{t^2}\quad \frac{K\ \ w}{t}\quad K_Fw\quad 0 \qquad (1)$$

where ^{2} is the two dimensional Laplacian operator in Cartesian coordinates w is transverse deflection, v is the Poisson ratio and D is the flexural rigidity at any point of the plate, is the mass density per unit volume h is thickness of plate, t is the time, K is damping constant and K_F is the foundation modulus. After changing the above equation to polar coordinates and

introducing the non dimensional variables, and if thickness of plate

H, modulus of elasticity and mass density are assumed to vary in the form,

$$H\quad H_0(1\quad X^2)$$
$$\overline{E}\quad E_0(1\quad X)$$
$$\overline{\ }\quad (1\quad X)$$

where is taper constant, is non homogeneity parameter one obtains,

$$+\left[\left\{\ 12\ \ X(1\quad X^2)^2\quad \frac{(2\quad v)(1\quad X^2)^3}{X}\right\}\ \right]\frac{^2W}{X^2}$$

$$\left[\left\{\ 6\ \ (1\quad X^2)^2\quad 24\ \ ^2X^2(1\quad X^2)\ \ \frac{(1\quad X^2)^3}{X^2}\quad 6\ \ X\frac{(2\quad v)(1\quad X^2)^2}{X}\right\}(1\quad X)\right]\frac{^2W}{X^2}$$

$$\left[\left\{\ 12\alpha\ Xv\frac{(1\quad \alpha X^2)^2}{X}\quad \frac{(1\quad \alpha X^2)^3}{X}\right\}\beta\right]\frac{\partial W}{\partial X}$$

$$\left[\left\{\ 6\quad \frac{(1\quad X^2)^2}{X}\quad 24\ {}^2X\ (1\quad X^2)^2\ \ 6\ \frac{(1\quad X^2)^2}{X}\quad \frac{(1\quad X^2)^3}{X^3}\right\}(1\quad X)\right]\frac{W}{X}$$

$$\frac{(1\quad X^2)^3\ (1\quad X)\ (1\quad {}^2)a^2\quad {}_0}{\overline{E_0}\ \ I^*}\frac{{}^2W}{t^2}\quad \frac{F_D}{C^*}\frac{W}{t}\quad \frac{F_p}{C^*}W\quad 0 \tag{2}$$

where

$$F_p\quad \frac{K_F}{E_0},\ F_d\quad \frac{K_Da}{E_0},C^*\quad \frac{H_0^3}{12(1\quad \mathrm{v}^2)},I^*\quad \frac{H_0^2}{12}$$

SOLUTION FOR DAMPED HAMONIC VIBRATIONS

For Damped Harmonic Vibrations, the solution is given by

$$W\quad (X,t)\quad \overline{W}(X)e^{\ \ t}\cos pt$$

Substituting (3) in (2) & solving we get,

$$(1\quad \beta X)^2(1\quad \alpha X^2)^4\frac{\partial^4\overline{W}}{\partial X^4}\quad 2\left[(1\quad \alpha X^2)^4\beta(1\quad \beta X)\ \left\{\ 6\alpha X(1\quad \alpha X^2)^3\quad \frac{(1\quad \alpha X^2)^4}{X}\right\}(1\quad \beta X)^2\right]\frac{\partial^3\overline{W}}{\partial X^3}$$

$$\left[\left\{\ 12\alpha X(1\quad \alpha X^2)^3(1\quad \beta X)\quad \frac{(2\quad \upsilon)(1\quad \alpha X^2)^4(1\quad \beta X)}{X}\right\}\beta\right]\frac{\partial^2\overline{W}}{\partial X^3}$$

$$\left[\left\{\ 6\alpha(1\quad \alpha X^2)^3\quad 24\alpha^2X^2(1\quad \alpha X^2)^2\quad \frac{(1\quad \alpha X^2)^4}{X^2}\quad 6\alpha\,(2\quad \upsilon)(1\quad \alpha X^2)^3\right\}(1\quad \beta X)^2\right]\frac{\partial^2\overline{W}}{\partial X^2}$$

$$\left[\left\{\ 12\alpha\upsilon(1\quad \alpha X^2)^3\quad \frac{(1\quad \alpha X^2)^4}{X}\right\}\beta(1\quad \beta X)\right]\frac{\partial\overline{W}}{\partial X}$$

$$\left[\left\{\quad 6\alpha\upsilon\frac{(1\quad\alpha X^2)^3}{X}\quad 24\alpha^2X\upsilon(1\quad\alpha X^2)^2\quad 6\alpha\frac{(1\quad\alpha X^2)^3}{X}\quad\frac{(1\quad\alpha X^2)^4}{X^3}\right\}(1\quad\beta X)^2\right]\frac{\partial\overline{W}}{\partial X}$$

$$\left[(1\quad X^2)^2(1\quad X)^2M^{*}\quad{}^2\quad D_KM^{*2}\quad(1\quad X)(1\quad X^2)E_FC^{*}\right]\overline{W}\quad 0$$

where,

$$D_K\quad\frac{3(1\quad{}^2)K^2}{{}_0E_0},\ M^*\quad\frac{1}{H_0^2},\ C^*\quad\frac{1}{H_0^3},\ E_F\quad\frac{12K_F(1\quad{}^2)}{E_0}$$

$$_2\quad\frac{12(1\quad{}^2)\ {}_0a^2p^2}{E_0}$$

Here p is circular frequency, D_k is damping parameter, is frequency parameter & E_F is a foundation Modulus parameter.

A Series solution is assumed in the form $\overline{W}(X)\quad\underset{0}{\ }\ a\ X^{C}\ ,\ a_0\quad 0$, where c is exponent of singularity. Applying method of Frobenius we get the values of $c = 0, 0, 2,2$ and we obtain solution for $\overline{W}$ corresponding to $c = 0$, as,

$$\overline{W}\quad a_0\left[1\quad\underset{3}{\ }\ A\ \ X\right]\quad a_2\left[X^2\quad\underset{3}{\ }\ B\ X\right]\tag{5}$$

In the above solution, we see that no new solution will arise corresponding to the other values of c, as they are already contained in the solution given by equation (5). Also here values of A and B for are functions of , ,D_k, E_F, M^*, N^* and .

BOUNDARY CONDITIONS & FREQUENCY EQUATIONS

The frequency equations for clamped amd simply suppiorted circular plates have been obtained by employing the appropriate boundary conditions.

Clamped plate:

For a circular plate clamped at edges $r = a$, the deflection W and slope of the plate element at edges should be zero.

Or

$$\overline{W}\big|_{X\quad a}\qquad \frac{\overline{W}}{X}\Big|_{X\quad 1}\qquad 0 \tag{6}$$

Using equation (5) and applying the boundary condition, (6) one obtains

$$a_0\left[1\qquad A_{\quad 3}\right]\quad a_2\left[1\qquad B_{\quad 3}\right]\qquad 0$$

$$a_0\left[\quad{}_{3}\quad A\right]\quad a_2\left[2\qquad {}_{3}\quad B\right]\qquad 0$$

Elimination the unknown constants a_0 and a_2, one obtains the frequency equation for (C-plate) as

$$\begin{vmatrix} V_1(\quad) & V_2(\quad) \\ V_3(\quad) & V_4(\quad) \end{vmatrix}\qquad 0 \tag{7}$$

where as,

$$V_1(\quad)\quad 1\qquad {}_{3}\quad A\ ,$$

$$w(r,t)\big|_{r\quad a}\qquad \frac{w(r,t)}{t}\Big|_{r\quad a}\qquad 0$$

$$V_2(\quad)\quad 1\qquad {}_{3}\quad B\ ,$$

$$V_3(\quad)\qquad {}_{3}\quad A\ ,$$

$$V_4(\quad)\quad 2\qquad {}_{3}\quad B\ ,$$

SIMPLY SUPPORTED PLATE

For a circular plate simply supported at the edge $r = a$, the deflection w and the moments M_r at the edge should be zero.

$$w\,(r,t)\big|_{r \quad a} \qquad M_r(r,t)\big|_{r \quad a} \qquad 0$$

Or,

$$\overline{W} \qquad \left[\frac{^2\overline{W}}{X^2} \quad \frac{\quad}{X}\frac{\overline{W}}{X}\right]_{X \quad 1} \qquad 0 \tag{8}$$

Applying these boundary conditions on the equation (5),one obtains

$$a_0\left[1 \qquad {}_3 \; A\right] \; a_2\left[1 \qquad {}_3 \; B\right] \quad 0$$

$$a_0\left[\quad {}_3 \quad (\qquad 1)A\right] \; a_2\left[2(1 \qquad) \qquad {}_3 \quad (\qquad 1)B\right] \quad 0$$

Elimination the unknown constants a_0 & a_2 one gets the frequency equation for simply supported plate as,

$$\begin{vmatrix} V_1(\;\;) & V_2(\;\;) \\ V_5(\;\;) & V_6(\;\;) \end{vmatrix} \quad 0 \tag{9}$$

where

$$V_5(\;\;) \qquad {}_3 \quad (\qquad 1)A \;,$$

$$V_6(\;\;) \quad 2(1 \qquad) \qquad {}_3 \quad (\qquad 1)B \;,$$

DEFLECTION FUNCTIONS & MOMENTS

The Transverse displacement function $\overline{W}$ (5) is given by:

$$\overline{W} \quad a_0\left[1 \qquad {}_3 \; A \; X\right] \; a_2\left[X^2 \qquad {}_3 \; B \; X\right] \tag{10}$$

The unknown constants a_0 and a_2 are determined by applying boundary condition at $X = 1$. The equation thus obtained are such that only the ratio of the constants can be determined. We take $a_0 = 1$, and get the value of a_2 as,

$$a_2 \quad \frac{1 \quad A_3}{1 \quad B_3}$$

Substituting a_0 and a_2 the above equation (10) becomes.

$$\overline{W} \quad \left[1 \quad A_3 X\right] \frac{1 \quad A_3}{B_3}\left[X^2 \quad B_3 X\right] \tag{11}$$

Also,

$$M_r \quad D\left[\frac{{}^2\overline{W}}{r^2} \quad \frac{}{r}\frac{\overline{W}}{r}\right]$$

Again enforcing the boundary condition $W = 0$ at $X = 1$ and sdopting the same value of a_0 and a_2 the non dimensional parameter is obtained in the from

$$\overline{M} \quad \frac{M_X}{D_0} \quad (1 \quad X^2)^3\left[{}_3 \quad (\quad 1)A \; X \quad {}^2\right] \tag{12}$$

$$(1 \quad X^2)^3 \frac{A_3}{B_3}\left[2(1 \quad) \quad {}_3 \quad (\quad 1)B \; X \quad {}^2\right]$$

where

$$D_0 \quad \frac{EH_0^3 a^3}{12(1 \quad {}^2)} \tag{13}$$

The values of for both edge conditions have been taken from equation (7) and (9).

RESULTS & DISCUSSION

All the numerical results for a Damped isotropic non homogenous circular plate of linearly varying thickness considering foundation effect corresponding to first two modes of vibration for both clamped & simply supported edge condition have been computed using latest computer techniques. The result have been tabulated and graphically shown. Poisson ratio has been taken to be 0.3 and thickness of plate 0.1. Terms of the series up to an accuracy of 10^{-8} in their absolute values have been retained. By allowing damping parameter, non homogeneity parameter and elastic foundation to be zero, the problem of homogenous circular plate of parabolically varying thickness the result so obtained compare well with already published work of Gupta [4].

Figures (1) and (2) shows the variation of frequency parameter ( ) with taper constant ( ) for different values of Non homogeneity parameter, damping parameter & elastic foundation parameter for both clamped plate as well as simply supported plate. We see that on increasing the value of taper constant there is gradual decrease in frequency parameter in both in both of modes. Figures (3) and (4) shows the variation of frequency parameter ( ) with foundation parameter (E_f) for different values of taper constant, non-homogeneity parameter & damping. There is increase in frequency parameter as we increase the foundation effect. This effect is greater for first mode than the second mode in both clamped plate & simply supported plate.

Figures (5) and (6) shows the variation of frequency parameter ( ) with damping parameter (D_k) for different values of non homogeneity parameter, taper constant and Elastic foundation parameter. We observe that as we increase the damping effect the frequency parameter decreases. The effect of damping on first mode is more significant than for second mode in both the case i.e. clamped plate as well as for simply supported plate. Figures (7) and (8) shows variation of frequency parameter ( ) with non homogeneity parameter ( ) considering fixed value of taper constant Damping parameter, and foundation parameter. Deflection parameter & moment parameter have also been computed for both edge conditions corresponding to the first two modes of vibration. Figures (9) and (10) show the variation of deflection function & Moment parameter with respect to the different points on the plate surface from axis of symmetry.

H_0=0.1, v=0.3, D_k= 0.0, E_f= 0.0, =0.0

Fig.1: Variation of ( ) a damped non-homogeneous circular plate of parabolically varying thickness resting on elastic foundation for different values of taper constant.

H_0=0.1, v=0.3, D_k= 0.1, E_f= 0.1, =0.1

Fig- 2: Variation of ( ) a damped non-homogeneous circular plate of parabolically varying thickness taper constant.

H_0=0.1, v=0.3, D_k= 0.0, = 0.0, =0.0

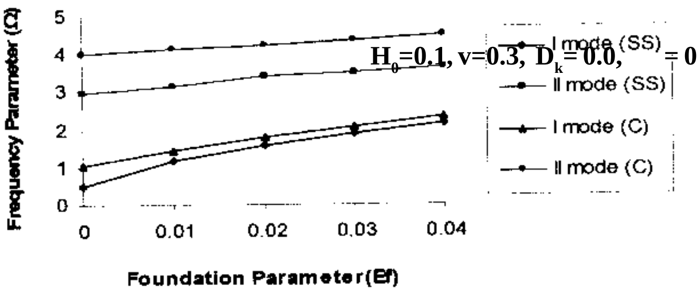


Fig- 3: Variation of ( ) a damped non-homogeneous circular plate of parabolically varying thickness resting on elastic foundation for different values of foundation parameter.

H_0=0.1, v =0.3, D_k= 0.1, = 0.1, =0.1

Fig- 4: Variation of ( ) of a damped non-homogeneous circular plate of parabolically varying thickness resting on elastic foundation for different values of foundation parameter.

H_0=0.1, v =0.3, E_f= 0.0, = 0.0, =0.0

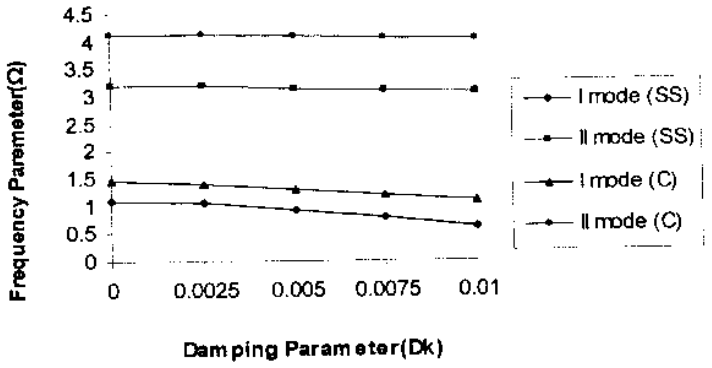


Fig- 5: Variation of ( ) of a damped non-homogeneous circular plate of parabolically varying thickness resting on elastic foundation for different values of damping parameter.

H_0=0.1, v =0.3, E_f= 0.1, = 0.1, =0.1

Fig- 6: Variation of ( ) of a damped non-homogeneous circular plate of parabolically varying thickness resting on elastic foundation of different values of damping parameter.

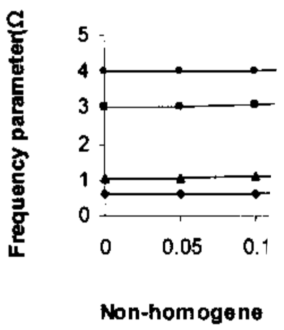
H_0=0.1, v =0.3, E_f= 0.1, = 0.1, D_k=0.1

Fig- 7: Variation of ( ) of a damped non-homogeneous circular plate of parabolically varying thickness resting on elastic foundation of different values of non-homogeneity parameter.

H_0=0.1, v =0.3, E_f= 0.1, = 0.1, D_k=0.1

Fig- 8: Variation of ( ) of a damped non-homogeneous circular plate of parabolically varying thickness resting on elastic foundation of different values of non-homogeneity parameter.

H_0=0.1, v =0.3, = 0.1, =0.1, D_k =0.01, E_f= 0.01



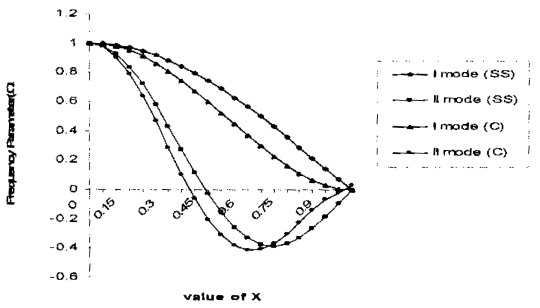


Fig- 9: Transverse Deflection at different point of a Non-homogeneous damped Circular Plate of Parabolically varying thickness resting on elastic foundation.

H_0=0.1, v =0.3, = 0.1, =0.1, D_k =0.01, E_f= 0.01

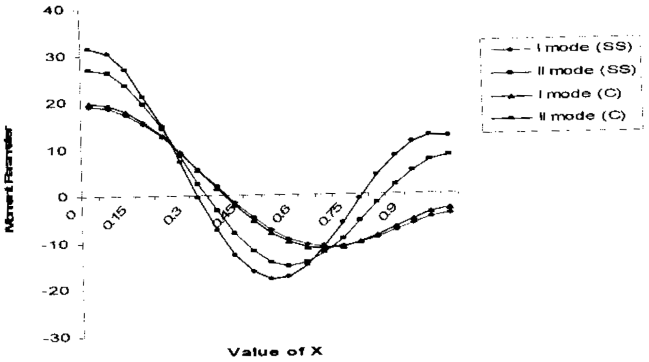


Fig- 10: Moment parameter (M) at different point of a No-homogeneous damped Circular Plate of Parabplically varying thickness resting on elastic foundation.

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A COMMON FIXED POINT THEOREM FOR A SEQUENCE OF MAPPINGS IN MENGER SPACES

R.C. Dimri*, N.S. Gariya* and Amit Singh*

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ABSTRACT

In the present paper, we prove a unique common fixed point theorem for a sequence of mappings in Menger spaces. Our result improves and modifies a result of B.S. Choudhary, in the sense that, the commutativity of the mappings has not been considered and contraction condition taken in a different way.

Keywords: sequence of mappings, Menger spaces, common fixed point.

AMS Subject Classification (2000): 54H25, 47H10, 54E70.

INTRODUCTION AND PRELIMINARIES

Sehgal [5] initiated the study of contraction mappings on probabilistic metric spaces (PM-spaces) (cf. also [6] and [7]). Since then many fixed point results for single and multivalued mappings on PM-spaces are obtained. Recently Choudhary [1] has established a unique common fixed point theorem for a sequence of mutually contractive and commuting sequences of self-mappings on Menger spaces. In the present paper we modify and improve the result of Choudhary [1] for a sequence of self-mappings under a different contraction condition and not using the commutativity of mappings.

We now recall some basic definitions and results. A mapping F: R⟶ [0, 1] is called a distribution function if it is non-decreasing, left continuous $\underset{x \quad R}{\text{sup}}$ F(x) = 0 and $\underset{x \quad R}{\text{sup}}$ F(x) = 1

$(d) F_{u,w}(x) F_{v,u} 1$

DEFINITION 1 [2]

A PM-space is an ordered pair (X,$\mathcal{F}$) where *X* is a nonempty set of elements and $\mathcal{F}$ is a mapping from *X* × *X* to £, the collection of all distribution functions. The value of $\mathcal{F}$ at (*u*, *v*) ∈ *X* × *X* is represented by $F_{u,v}$. The functions $F_{u,v}$ are assumed to satisfy the following conditions:

$(a) F_{u,v}(x) \quad 1$ for all *x* > 0 iff *u* = *v*;

$(b) F_{u,v}(0) \quad 0;$

and $F_{v,w}(y) \quad 1$ then $F_{u,W}(x \quad y) \quad 1$

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*Department of Mathematics, HNB Garhwal University, , Srinagar 246174 E-mail:dimrirc@gmail.com, singhamit841@gmail.com

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ON EINSTEIN-KAEHLERIAN CONHARMONIC* RECURRENT SPACES

A.K. Singh*, U.S. Negi* and Aparna Rawat*

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ABSTRACT

Mathai [2] and Walker [7] have studied Kaehlerian recurrent spaces and Ruse's spaces of recurrent curvature respectively, Singh and Singh [4] have defined and studied Kaehlerian conharmonic recurrent and Kaehlerian conharmonic symmetric spaces and several theorems have been established. Further, Singh [6] has defined and studied Einstein-Kaehlerian conharmonic recurrent spaces and several theorems have been investigated, Singh [11] has defined and studied Einstein-Kaehlerian projective recurrent spaces and several theorems have been obtained. Also, Singh and Kumar [13] have defined and studied on Einstein-Tachibana conharmonic recurrent spaces and several theorems have been established. Later, Chauhan, Chauhan and Singh [14] have defined and studied on Einstein-Kaehlerian space with recurrent Bochner curvature tensor and several theorems have been obtained.

Here we define and study Einstein-Kaehlerian conharmonic* recurrent spaces and several theorems have been derived. The necessary and sufficient conditions for an Einstein-Kaehlerian conharmonic* recurrent space to be Kaehlerian recurrent and Einstein-Kaehlerian conharmonic recurrent have been investigated.

Keywords: Einstein-Kaehlerian Conharmonic*, Conharmonic, Bochner, Projective, Recurrent, Space.

Classification Number : 53Bxx, 32C15, 46A13, 46M40, 53B35, 53C55.

INTRODUCTION

An n (= $2m$) dimensional Kaehlerian space in a Riemannian space which admits a structure tensor field F^h_i satisfying :

$$F^h_j F^i_h = - \ {}^i_j, \tag{1.1}$$

$$F_{ij} = - F_{ji}, \ (F_{ij} = F^a_i) \tag{1.2}$$

and

$$F^h_{i,j} == 0 \tag{1.3}$$

where the comma (,) followed an index denotes the operator of covariant differentation with

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*Department of Mathematics, HNB Garhwal University, Campus Badshahithaul, Tehri Garhwal – 249199 (Uttarakhand), India.

respect to the metric tensor g_{ij} of the Riemannian space.

The Riemannian curvature tensor field is defined by

$$R^h{}_{ijk} = \quad {}_i\{{}^{\,h}_{l\;k}\} - \quad {}_j\{{}^{\,h}_{i\;k}\} - \{{}^{\,h}_{i\;l}\}\{{}^{\,l}_{j\,k}\} - \{{}^{\,h}_{j\;l}\}\{{}^{\,l}_{i\,k}\}, \tag{1.4}$$

whereas

$$R_{ij} = R^h{}_{hij} \; and \; R = R_{ij}\, g^{ij}.$$

The Ricci tensor and the scalar curvature are respectively.

The Kaehlerian conharmonic curvature tensor $T^h{}_{ijk}$, is given by [3] :

$$T^h{}_{ijk} = \mathrm{R}^{\mathrm{h}}{}_{\mathrm{ijk}} +$$

$$+ \quad S_{ik}F^h_j \quad S_{jk}F^h_i \quad F_{ik}S^h_j \quad F_{jk}S^h_i \quad 2S_{ij}\ F^h_k \quad 2F_{ij}S^h_k) \tag{1.5}$$

And, the Kaehlerian chonarmonic* curvature tensor $*\, T^h{}_{ijk}$ is given by [8]

$$*T^h{}_{ijk} \;\underline{\underline{def}} \tag{1.6}$$

where $S_{ij} = F^a{}_i\, R_{aj}$.

Let us suppose that a kaehlerian space is an Einstein one, then the Ricci tensor satisfies

$$R_{ij} == R/n \;\; g_{ij},\, R,_a == 0\,. \tag{1.7}$$

from which, we have

$$R_{ij,a} == 0,\, S_{ij,a} == 0, \text{ and } S_{ij} == R/n\, F_{ij}\,. \tag{1.8}$$

If a Kaehlerian space is an Einstein one, the Kaehlerian conharmonic curvature tensor and Kaehlerian conharmonic* curvature tensor reduce to.

$$E^h_{ijk} \quad R^h_{ijk} \quad \frac{2R}{n(n \quad 4)}(g_{ik}\delta^h_j \quad g_{jk}\delta^h_i \quad F_{ik}F^h_j \quad F_{jk}F^h_i \quad 2F_{ik}F^h_k)\ . \tag{1.9}$$

and

$$**T^h_{ijk} \qquad R^h_{ijk} \quad \frac{R}{n(n \quad 2)}(g_{ik} \;\; {}^h_j \quad g_{jk} \;\; {}^h_i) \tag{1.10}$$

respectively.

In view of equations (1.9) and (1.10), we obtain

$$E^{h}_{ijk} \quad {}^{**}T^{h}_{ijk} \quad \frac{(n \quad 8)R}{n(n \quad 2)(n \quad 4)}(g_{ik} \quad {}^{h}_{j} \quad g_{jk} \quad {}^{h}_{i})$$

$$\frac{2R}{n(n \quad 4)}(F_{ik}F^{h}_{j} \quad F_{jk}F^{h}_{i} \quad 2F_{ij}F^{h}_{k}) \tag{1.11}$$

We shall use the following:

Definition (1.1) : A Kaehler space is said to be recurrent, if we have [1])

$$R^{h}_{ijk,a} - \quad {}_{a}R^{h}_{ijk} = 0, \tag{1.12}$$

For some non -zero recurrence vector $_{a}$ and is called Kaehlerian Ricci- recurrent, if it satisfies the relation

$$R_{ij,a} - \quad {}_{a}R_{ij} = 0, \tag{1.13}$$

Multiplying the above equation by g^{ij}, we get

$$R,_{a} - \quad R = 0. \tag{1.14}$$

Remark (1.1) : From (1.13), it follows that every Kaehlerian recurrent space is Kaehlerian Ricci -recurrent but the converse is not necessarily true.

Definition (1.2) : A Kaehler space satisfying the relation [6]

$$E^{h}_{ijk,a} \quad {}_{a}E^{h}_{ijk} \quad 0 \tag{1.15}$$

where $\quad$ is a non-zero recurence vector, will be called an Einstein-Kaehlerian conharmonic recurrent space or briefly an E-K * space.

Definition (1.3): A Kaehler space is called a space of constant holmorphic sectional curvature tensor. If the tensor U^{h}_{ijk} defined by [5].

$$U^{ah}_{ijk} \quad R^{h}_{ijk} \quad \frac{R}{n(n \quad 2)}(g^{a}_{ik} \quad {}^{h}_{j} \quad g_{jk} \quad {}^{h}_{i} \quad F_{ik}F^{h}_{j} \quad F_{jk}F^{h}_{i} \quad 2F_{ij}F^{h}_{k}) \tag{1.16}$$

vanishes identically.

EINSTEIN- KAEHLERIAN CONHARMONIC* RECURRENT SPACES

Defination (2.1) : A Kaehler space satisfying the relation:

$$**T^h_{ijk,a} \qquad {}_a **T^h_{ijk} \quad 0 \tag{2.1}$$

where ${}_a$ is a non -zero recurrence vector, will be called an Einstein- Kaehlerian conharmonic* recurrent space or briefly an *E-T** space.

We have the follwing:

Theorem (2.1): A necessary and sufficient condition for an *E-T** space to be a Kaehlerian recurrent is that the scalar curvature be equal to zero.

Proof: Suppose that an *E-T** Space is Kaehlerian recurrent, making use of equations (1.7), (1.8) and (1.10) in (2.1), we get

(2.2)

Since, an *E-T** space is Kaehlerian recurrent, equation (2.2) reduces to

$$\frac{R}{n(n \quad 2)}(g_{ik} \ \delta^h_j \quad g_{jk}\delta^h_i) \quad 0 \tag{2.3}$$

which gives $R = 0$

Conversely, if an *E-T** space satisfied $R = 0$, then equation (2.2) reduce to

$$R^h_{ijk,a} \qquad {}_a R^h_{ijk} \quad 0$$

which shows that the space is Kaehlerian recurrent. This completes the proof.

Similary, is view of Theorem (2.1) and equations (1.7), (1.8), (1.9) and (1.15), we can prove the following:

Theorem (2.2): A necessary and sufficient condition for an *E-K** space to be Kaehlerian recurrent is that the scalar curvature be equal to zero.

Theorem (2.3): An *E-T** space is an *F-K** space iff the scalar curvature R is equal to zero.

Proof : Suppose that an *E-T** space is a an *E-K** space. Differentiating (1.11) covariantly with respect x^a and using (1.7) we obtain

$$E^h_{ijk,a} \qquad **T^h_{ijk,a} \tag{2.4}$$

Multiplying (1.11) by ${}_a$ and subtracting from (2.4), we have

)

$$\frac{2\quad {}_aR}{n(n\quad 4)}(F_{ik}F_j^h\quad F_{jk}F_i^h\quad 2F_{ij}F_k^h) \tag{2.5}$$

Now, making use of the above supposition, equation (2.5) reduces to

$$\lambda_a R\,[\frac{(n\quad 8)}{n(n\quad 2)(n\quad 4)}(g_{ik}\ \delta_j^h\quad g_{jk}\delta_i^h)\quad \frac{2}{n(n\quad 4)}\left(F_{ik}F_j^h\quad F_{jk}F_i^h\quad 2F_{ij}F_k^h\right)]\quad 0$$

which implies that $R = 0$.

Conversely, let us suppose that in an *E-T** space, the scalar curvature $R = 0$. Hance equation (2.5) reduces to

$$E^h_{ijk,a}\qquad {}_aE^h_{ijk}\qquad 0$$

which shows that the space is an *E-K** space.This completes the proof.

Theorem (2.4) : If a Kaehler space satisfies any two of the following:

(i) the space is an *E-K** space,

(ii) the space is an *E-T** space,

(iii) the scalar curvature is equal to zero,

It must also satisfy the third.

Proof: An *E-K** and an *E-T** spaces are characterized respectively by equations (1.15) and (2.1).

The statement of the theorem follows in view of equations (1.15), (2.1) and (2.5).

We have the following lemmas from Walker [7], and Yano and Bochner [10].

$a\quad b\qquad a\quad b\qquad a\quad b\qquad 0$

Lemma (2.1): The curvature tensor R_{hijk} satisfies the identity

$$R_{hijk,lm} - R_{hijk,ml} + R_{jklm,hi} - R_{jklm,ih} + R_{lmhi,jk} - R_{lmhi,kj} = 0 \tag{2.6}$$

where $R^h{}_{hijk,l,m} \overset{def}{=\!=} R^h{}_{hijk,lm}$

Lemma (2.2): If a ,b are quantities satisfying

$$a\qquad a\qquad \text{and} \tag{2.7}$$

For , , =1, 2, ---------, N, then either all the a are zero or all the b are zero Making use of the above Lemmas, we shall prove the following :

Theorem (2.5): In an E-T^* space, either recurrence vector is gradient or the space is of constant holomorphic sectional curvature.

Proof: Differentiating (2.2) covariantly with respect to x^b and using equations (1.3), (1.7), (1.8) and (1.10) we get

$$R^h{}_{ijk,ab} \qquad (\quad {}_{a,b} \qquad {}_a \quad {}_b) {**}T^h_{ijk} \qquad (2.8)$$

where $R^h{}_{ijk,a,b} \overset{def}{=\!=} R^h{}_{ijk,ab}$.

Multiplying (2.8) with $g_{hl,}$ we obtain

$$R_{ijkl,ab} == (\ddot{e}_{a,b} + \ddot{e}_a \ddot{e}_b) {**}T_{ijkl}. \qquad (2.9)$$

From (2.9) and the identity (2.6), we get

$$\ddot{e}_{ab} {**}T_{ijkh} + \ddot{e}_{ij} {**}T_{khab} + \ddot{e}_{kh} {**}T_{abij} = 0. \qquad (2.10)$$

where $\ddot{e}_{ab} \overset{def}{\quad} \ddot{e}_{b,a} - \ddot{e}_{a,b}$.

Equation (2.10) is of the form (2.7), since ${**}T_{ijkh} == {**}T_{khij}$.

Thus, from Lemma (2.2), we have Theorem (2.5).

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ON THE FOUNDATION OF THE ADOMIAN DECOMPOSITION METHOD

Fawzi Abdelwahid [*] and Randolph Rach[**]

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ABSTRACT

In this work, we review the Adomian decomposition method for the case of a nonlinear differential equation with a general operator involving a special parameter λ., and examine the Taylor series expansion of a nonlinear term represented by an analytic function. For this type of operator, this study shows that Adomian's approach is a special choice corresponding to $\lambda=1$. This study also defines the Adomian polynomials and explains Adomian's choices. As a result of this study, we feature Abdelwahid's Algorithm for generating the Adomian polynomials for different types of nonlinear terms, which is eminently suitable for symbolic programming by exploiting the sifting property of the well-known Kronecker delta function. Furthermore this procedure can be used to calculate the Adomian polynomials for any nonlinear term as represented by an analytic function. Finally this study indicates that the case of $\lambda \neq 1$ presents an interesting new challenge for future research and studies including the case of $\lambda = \lambda(x)$ and introduces a general approach for the Adomian decomposition method.

Keywords: Adomian decomposition method; Adomian polynomials; Adomian decomposition series; Adomian's Integral Equation; parameterized Taylor expansion series; analytic parameter, nonlinear deterministic differential equations

INTRODUCTION

Adomian decomposition method [1-7] is a significant, powerful method, which provides an efficient means for the analytic and numerical solution of differential equations, which model real-world physical applications. In recent years Adomian and others have successfully applied the decomposition method to ordinary, partial, delay [8-9], and non-integer order [10] differential equations for a wide class of nonlinearities, including polynomial, exponential, trigonometric, composite, negative power [11], radical [12] and even decimal power [13] nonlinearities. Adomian's decomposition method allows us to solve nonlinear differential equations without having to appeal to the decidedly questionable practices of perturbation or linearization. Furthermore Adomian et al. [14-15] have developed a new approach to numerical integration algorithms based upon Adomian's decomposition method.

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[*] University of Garyounis, Faculty of Science, Department of Mathematics, Benghazi, Libya fawziabd@hotmaiL.com

[**] 316 South Maple Street, Hartford, Michigan 49057-1225, U. S. A. tapstrike@triton.net